Interaction between Noncommutative Open String and Closed-String Tachyon

Akira Kokado[*], Gaku Konisi[†] and Takesi Saito[‡]

Kobe International University, Kobe 655-0004, Japan[*]

Department of Physics, Kwansei Gakuin University, Nishinomiya 662-8501, Japan[†,‡]

March 2001

Abstract

We construct a vertex operator which describes an emission of the ground-state tachyon of the closed string out of the noncommutative open string. Such a vertex operator is shown to exist only when the momentum of the closed-string tachyon is subject to some constraints coming from the background B field. The vertex operator has a multiplicative coupling constant $g(\sigma)$ which depends on σ as $g(\sigma) = \sin^2 \sigma$ in $0 \leq \sigma \leq \pi$. This behavior is the same as in the ordinary $B = 0$ case.

[*]E-mail: kokado@kobe-kiu.ac.jp
[†]E-mail: konisi@kwansei.ac.jp
[‡]E-mail: tsaito@yukawa.kyoto-u.ac.jp

I. Introduction

The concept of a quantized spacetime was first proposed by Snyder [1], and has received much attention over the past few years [2-6]. Especially interesting is a model of open strings propagating in a constant antisymmetric B field background. Previous studies show that this model is related to the noncommutativity of D-branes [7,8], and in the zero slope limit to noncommutative gauge theories [5].

In the present paper we would like to construct a vertex operator which describes an emission of the ground-state tachyon of the closed string out of the noncommutative open string. Such a vertex operator has been considered in the ordinary open string theory [9], and for the emission of a graviton in the literatures [10],[11]. Let us call such a tachyon the closed-string tachyon. Contrary to the open-string tachyon, the closed-string tachyon can be emitted from any point of the open string. The vertex operator contains a multiplicative coupling constant $g(\sigma)$, which depends on σ as $g(\sigma) = \sin^2\sigma$ in $0 \leq \sigma \leq \pi$. When the constant antisymmetric B field is present, the open string becomes noncommutative at both end-points. In this case we will find that such a vertex operator exists only when the momentum of the closed-string tachyon is subject to some constraints coming from the background B field.

In Sec.II the model of noncommutative open string is summarized. In Sec.III we construct the vertex operator for the closed-string tachyon. The final section is devoted to concluding remarks.

II. Noncommutative open string

In order to fix notations let us summarize the model of bosonic open string propagating in a constant antisymmetric B field background. In the conformal gauge the world sheet action of the open string is

$$S = -\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma[\eta_{\mu\nu}\partial_\alpha X^\mu \partial^\alpha X^\nu - \epsilon^{\alpha\beta}B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu], \tag{2.1}$$

where $\eta_{00} = -1$,Σ is an oriented world-sheet with boundary (signature (-1, 1)) and $B_{\mu\nu}$ is assumed to be constant. For simplicity we have set $2\alpha' = 1$. The equation of motion and boundary conditions follow from this action

$$\partial_\alpha\partial^\alpha X^\mu = 0, \tag{2.2}$$

$$\eta_{\mu\nu}X'^\nu + B_{\mu\nu}\dot{X}^\nu|_{\sigma=0,\pi} = 0. \tag{2.3}$$

We are looking at a D25-brane with the constant B field. From (2.2) and (2.3) one obtains the following solution

$$X^\mu(\tau,\sigma) = q^\mu + a_0^\mu\tau + (\frac{\pi}{2} - \sigma)B^\mu{}_\nu a_0^\nu + \sum_{n\neq 0}\frac{1}{n}e^{-in\tau}(ia_n^\mu \cos n\sigma - B^\mu{}_\nu a_n^\nu \sin n\sigma). \tag{2.4}$$

The conjugate momentum is given by

$$
\begin{aligned}
P_\mu &= \frac{1}{\pi}(\eta_{\mu\nu}\dot{X}^\nu + B_{\mu\nu}X'^\nu) \\
&= \frac{1}{\pi}\sum_{n=-\infty}^{\infty} G_{\mu\nu}a_n^\nu e^{-in\tau}\cos n\sigma,
\end{aligned}
\tag{2.5}
$$

where

$$
G_{\mu\nu} = \eta_{\mu\nu} - B_\mu{}^\rho B_{\rho\nu}. \tag{2.6}
$$

According to the Dirac quantization for this constrained system[12],[13], we obtain the following commutation relations:

$$
\begin{gathered}
[X^\mu(\tau,\sigma), P_\nu(\tau,\sigma')] = i\delta^\mu{}_\nu\delta_c(\sigma,\sigma') \\
[P_\mu(\tau,\sigma), P_\nu(\tau,\sigma')] = 0, \\
[X^\mu(\tau,\sigma), X^\nu(\tau,\sigma')] = i\pi\theta^{\mu\nu}\{1-\epsilon(\sigma+\sigma')\},
\end{gathered}
\tag{2.7}
$$

where ϵ is the sign function, and the noncommutative parameter $\theta^{\mu\nu}$ is defined as

$$
\theta^{\mu\nu} = -B^\mu{}_\rho(G^{-1})^{\rho\nu}. \tag{2.8}
$$

From (2.7) one finds commutators for normal modes

$$
\begin{gathered}
[a_m^\mu, a_n^\nu] = m\delta_{m+n,0}(G^{-1})^{\mu\nu}, \\
[q^\mu, a_n^\nu] = i\delta_{n,0}(G^{-1})^{\mu\nu}, \\
[q^\mu, q^\nu] = 0.
\end{gathered}
\tag{2.9}
$$

Now let us write (2.4) as follows[1]:

$$
X(\tau,\sigma) = \frac{1}{2}[X_+(\tau+\sigma) + X_-(\tau-\sigma)],
$$

$$
\begin{aligned}
X_+ &= q + (a_0 - Ba_0)(\tau+\sigma) + \frac{\pi}{2}Ba_0 + i\sum_{n=0}\frac{1}{n}e^{-in(\tau+\sigma)}(a_n - Ba_n), \\
X_- &= q + (a_0 + Ba_0)(\tau-\sigma) + \frac{\pi}{2}Ba_0 + i\sum_{n=0}\frac{1}{n}e^{-in(\tau-\sigma)}(a_n + Ba_n).
\end{aligned}
\tag{2.10}
$$

In the following we use a complex number $z = e^{\tau+i\sigma}$. By the replacement $\tau \to -i\tau$ in (2.10) we have

$$
X_+(z) = q + \frac{\pi}{2}\sinh\beta\cdot\alpha_0 - ie^{-\beta}\alpha_0\ln z + i\sum_{n\neq 0}\frac{1}{n}z^{-n}e^{-\beta}\alpha_n,
$$

[1] There is an ambiguity in the zero mode when $X(\tau,\sigma)$ is divided into X_+ and X_-. However, there causes no effect in the result.

$$X_-(\bar{z}) = q + \frac{\pi}{2}\sinh\beta\cdot\alpha_0 - ie^{\beta}\alpha_0\ln\bar{z} + i\sum_{n\neq 0}\frac{1}{n}\bar{z}^{-n}e^{\beta}\alpha_n, \tag{2.11}$$

where β and α_n are defined by

$$B = \tanh\beta,$$
$$a_n = \cosh\beta\cdot\alpha_n. \tag{2.12}$$

The "metric" $G_{\mu\nu}$ in (2.6) is related to the "vielbein" $\xi = \cosh^{-1}\beta$ through equations

$$G = 1 - B^2 = 1 - \tanh^2\beta = \cosh^{-2}\beta = \xi\xi. \tag{2.13}$$

Commutation relations for α_n and q are

$$[\alpha_m, \alpha_n] = m\delta_{m+n,0},$$
$$[q, \alpha_0] = i\cosh\beta. \tag{2.14}$$

The Virasoro operator is then given by the energy-momentum tensor $T_\pm(z)$

$$L_n = \frac{1}{2\pi i}\oint dz z^{n+1}T_\pm(z) = \frac{1}{2}\sum :\alpha_k\alpha_{n-k}:,$$
$$T_\pm(z) = \frac{1}{2}:J_\pm(z)^2:, \tag{2.15}$$
$$J_\pm(z) = i\partial_z X_\pm(z) = \sum e^{\mp\beta}\alpha_n z^{-n-1}$$

and satisfies the same Virasoro algebra as in the ordinary open string theory without the B field.

III. Vertex operator for closed string tachyon

Let $V(z,\bar{z})$ be the vertex operator which describes the emission of the closed string tachyon out of the noncommutative open string. The Virasoro operator $\tilde{L}(f)$ of this interacting system is defined as

$$\tilde{L}(f) = \frac{1}{\pi}\int_0^\pi d\sigma(\tilde{T}_{00}f^0 + \tilde{T}_{01}f^1), \tag{3.1}$$

where $\tilde{T}_{\alpha\beta}$ is the energy-momentum tensor with

$$\tilde{T}_{00} = \frac{1}{2}:(\dot{X}^2 + X'^2): + V, \qquad \tilde{T}_{01} =: \dot{X}X':, \tag{3.2}$$

and f^α is expressed by an arbitrary function $f(\tau\pm\sigma)$ as

$$f^0 = \frac{1}{2}[f(\tau+\sigma) + f(\tau-\sigma)], \qquad f^1 = \frac{1}{2}[f(\tau+\sigma) - f(\tau-\sigma)]. \tag{3.3}$$

All operators are considered in the interaction picture. Equation (3.1) can be rewritten into the free Virasoro operator plus the vertex part

$$\tilde{L}(f) = L(f) + V(f), \tag{3.4}$$

where

$$L(f) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\sigma f(\tau+\sigma) : \frac{1}{2}(\dot{X} + X')^2 :,$$
$$V(f) = \frac{1}{\pi}\int_{0}^{\pi} d\sigma f^0(\tau,\sigma) V(\tau,\sigma). \tag{3.5}$$

Note that $V(\tau,\sigma)$ is not an even function of σ. In the complex number

$$L(f) = \frac{1}{2\pi i}\oint dz z f(z) T_{\pm}(z). \tag{3.6}$$

Here the integration path is a closed path around the origin.

The operator $\tilde{L}(f)$ should satisfy the Virasoro algebra

$$[\tilde{L}(f), \tilde{L}(g)] = i\tilde{L}(f \overleftrightarrow{\partial} g) + \text{anomaly term}. \tag{3.7}$$

This is equivalent to following equations:

$$[L(f), V(g)] - [L(g), V(f)] = iV(f \overleftrightarrow{\partial} g), \tag{3.8}$$

$$[V(f), V(g)] = 0. \tag{3.9}$$

In the following we look for the vertex operator which satisfies Eqs.(3.8) and (3.9). Let us assume

$$V(z, \bar{z}) = U_+(z) U_-(\bar{z}) \tag{3.10}$$

and

$$U_{\pm}(w) =: \exp\{ikX_{\pm}(w)\} : . \tag{3.11}$$

We then calculate the operator product expansion of $T_{\pm}(z)$ and $U_{\pm}(w)$. First we find the contraction

$$< X_{\pm}(z) X_{\pm}(w) >= -\ln(z-w) - (e^{\mp\beta}\cosh\beta - 1)\ln z + \text{const}. \tag{3.12}$$

If we define $\phi_{\pm}(w)$ by $\phi_{\pm}(w) = ikX_{\pm}(w)$, then it follows that

$$< J_{\pm}(z)\phi_{\pm}(w) >= \frac{1}{z-w}k + (e^{\mp\beta}\cosh\beta - 1)k\frac{1}{z}. \tag{3.13}$$

Since

$$< J_{\pm}(z) U_{\pm}(w) >=< J_{\pm}(z) : \exp\phi_{\pm}(w) :>=< J_{\pm}(z)\phi_{\pm}(w) > \frac{\partial U_{\pm}(w)}{\partial \phi_{\pm}(w)}, \tag{3.14}$$

one gets, dropping the $\pm$ suffices,

$$
\begin{aligned}
< T(z)U(w) > &= < \frac{1}{2} : J(z)J(z) : U(w) > \\
&= : J(z) < J(z)U(w) >: + \frac{1}{2} J(z)^{\bullet} J(z)^{\bullet\bullet} U(w)^{\bullet,\bullet\bullet} \qquad (3.15)\\
&= i\partial_z X(z)[\frac{1}{z-w}k + (e^{\mp\beta}\cosh\beta - 1)k\frac{1}{z}]\frac{\partial U(w)}{\partial\phi(w)} + \frac{1}{2}[***]^2 \frac{\partial^2 U(w)}{\partial\phi^2(w)}.
\end{aligned}
$$

The first $1/(z-w)$ term becomes

$$
\partial_z\phi(z)\frac{1}{z-w}\frac{\partial U(w)}{\partial\phi(w)} = \frac{1}{z-w}\partial_w U(w) + \text{regular term.} \qquad (3.16)
$$

The second $1/z$ term is regular around $z = w$. The third bracket squared-term reduces to

$$
[***]^2 = [\frac{1}{z-w}k - \bar{\beta}k\frac{1}{z}]^2 = \frac{k^2}{(z-w)^2} - 2\frac{k\bar{\beta}k}{z-w}\frac{1}{z} + \frac{k\bar{\beta}^2 k}{z^2}, \qquad (3.17)
$$

where

$$
\bar{\beta} \equiv 1 - e^{\mp\beta}\cosh\beta. \qquad (3.18)
$$

To sum up we have the operator product expansion

$$
T(z)U(w) = \frac{k^2/2}{(z-w)^2}U(w) + \frac{1}{z-w}\partial_w U(w) - \frac{k\bar{\beta}k}{z-w}\frac{1}{z}U(w) + \text{regular term.} \qquad (3.19)
$$

The third term in the right-hand side violates the conformal covariance of $U(w)$. So, in the following we will put a constraint for the momentum k

$$
k\bar{\beta}k = 0. \qquad (3.20)
$$

Thus we find that the vertex operator $U(w)$ has the conformal weight $h = k^2/2$. The equation (3.19) without the third term is equivalent to the equation

$$
[L(f), U(w)] = (k^2/2)\partial[wf(w)]U(w) + wf(w)\partial U(w). \qquad (3.21)
$$

When

$$
k^2 = 2, \qquad (3.22)
$$

the right-hand side becomes a total derivative $\partial[wf(w)U(w)]$. Compatibility of (3.22) with (3.20) will be discussed later. In this case one finds the equation

$$
[L(f), V(w,\bar{w})] = \partial_w[wf(w)V(w,\bar{w})] + \partial_{\bar{w}}[\bar{w}f(\bar{w})V(w,\bar{w})]. \qquad (3.23)
$$

In the real time formulation the right-hand side can be written as

$$
\begin{aligned}
-i\partial_\alpha[f^\alpha V] + 2f^0 V &= -i\partial_0[f^0 V] - i\partial_1[f^1 V] + 2f^0 V \\
&= -i\partial_0 f^0 V - if^0\partial_0 V - i\partial_1[f^1 V] + 2f^0 V \qquad (3.24)\\
&= -i\partial_1 f^1 V - i\partial_0 f^0 V - i\partial_1[f^1 V] + 2f^0 V,
\end{aligned}
$$

so that we obtain

$$[L(f), V(g)] = \frac{1}{\pi}\int_0^\pi d\sigma g^0\{-i\partial_\sigma f^1 V - if^0\partial_\tau V - i\partial_\sigma(f^1 V) + 2f^0 V\}.$$

Hence there holds

$$\begin{aligned}
&[L(f), V(g)] - [L(g), V(f)] \\
&\quad = -i\frac{1}{\pi}\int_0^\pi d\sigma g^0[\partial_\sigma f^1 V + \partial_\sigma(f^1 V)] - (f \leftrightarrow g) \\
&\quad = -i\frac{1}{\pi}\int_0^\pi d\sigma[g^0\partial_\sigma f^1 - f^1\partial_\sigma g^0]V - (f \leftrightarrow g) \\
&\quad = i\frac{1}{\pi}\int_0^\pi d\sigma[f^1\partial_\sigma g^0 + f^0\partial_\sigma g^1]V - (f \leftrightarrow g) \qquad (3.25) \\
&\quad = i\frac{1}{2\pi}\int_0^\pi d\sigma[(f\partial g)(\tau+\sigma) + (f\partial g)(\tau-\sigma)]V - (f \leftrightarrow g) \\
&\quad = iV(f \overleftrightarrow{\partial} g).
\end{aligned}$$

This assures Eq.(3.8).

Now let us discuss the compatibility of Eq.(3.20) with Eq.(3.22). The Eq.(3.18) can be rewritten as

$$\bar{\beta} = \pm e^{\mp\beta}\sinh\beta. \qquad (3.26)$$

Hence Eq.(3.20) is equivalent to the equation

$$B_{\mu\nu}k^\nu = 0. \qquad (3.27)$$

This gives $k_i = k_j = 0$ for i, j with $B_{ij} \neq 0$ for the canonical form. The on-shell condition $k^2 = 2$ holds only with the elements in the subspace $B_{ij} = 0$. [A possible four-dimensional example satisfying these two conditions is $(k_0, k_1, 0, 0)$ with ${k_1}^2 - {k_0}^2 = 2$ for $B_{01} = 0$, $B_{23} \neq 0$. The three-dimensional momentum $(k_1, 0, 0)$ turns out to be in the direction of the magnetic field $(B_1 \equiv B_{23}, 0, 0)$.] From the constraints (3.20) and (3.22) one can deduce

$$2 = ke^{\pm\beta}\cosh\beta\cdot k = ke^{\pm 2\beta}k = k\cosh 2\beta\cdot k. \qquad (3.28)$$

Let us then rewrite the vertex operator $V(z,\bar{z}) = U_+(z)U_-(\bar{z})$ into the normal product. Making use of the formulas (3.28) we have

$$\begin{aligned}
V(z,\bar{z}) &= :e^{ikX_+(z)}::e^{ikX_-(\bar{z})}: \\
&= \frac{(z-\bar{z})^2}{z\bar{z}} :e^{ik[X_+(z)+X_-(\bar{z})]}: = \frac{(z-\bar{z})^2}{z\bar{z}} :e^{2ikX(z,\bar{z})}: . \qquad (3.29)
\end{aligned}$$

Here, $K \equiv 2k$ corresponds to a momentum of an external field coupled to the noncommutative open string $X(z,\bar{z})$. Since $K^2 = (2k)^2 = 8$ (we have set $2\alpha' = 1$), this means that the external field is the ground-state tachyon of the closed string. The coupling constant factor $g(\sigma) = (z-\bar{z})^2/z\bar{z}$ is proportional to $\sin^2\sigma$. This behavior is the same as in the ordinary open string theory without the B field.

Finaly it still remains to check Eq.(3.9). This equation is true if vertex operators $V(z, \bar{z})$ and $V(w, \bar{w})$ are commutable with each other. In order to see this, let us note the following equations:

$$U_+(w)U_-(\bar{z}) = \frac{(w-\bar{z})^2}{w\bar{z}} : U_+(w)U_-(\bar{z}) := U_-(\bar{z})U_+(w), \tag{3.30}$$

$$U_\pm(z)U_\pm(w) = \frac{(z-w)^2}{zw} : U_\pm(z)U_\pm(w) := U_\pm(w)U_\pm(z). \tag{3.31}$$

By using these equations one can see

$$\begin{aligned} [V(z,\bar{z}), V(w,\bar{w})] &= [U_+(z)U_-(\bar{z}), U_+(w)U_-(\bar{w})] \\ &= U_+(z)[U_-(\bar{z}), U_+(w)]U_-(\bar{w}) + U_+(w)[U_+(z), U_-(\bar{w})]U_-(\bar{z}) \\ &= 0. \end{aligned} \tag{3.32}$$

This proves Eq.(3.9).

IV. Concluding remarks

We have drived the vertex operator (3.29) which describes an emission of the closed-string tachyon out of the noncommutative open string. Such a vertex operator has been shown to exist only when the momentum of the closed-string tachyon is subject to the constraints Eqs.(3.20) and (3.22). The vertex operator has a multiplicative coupling constant given by $g(\sigma) = \sin^2\sigma$, $0 \le \sigma \le \pi$. This behavior is the same as in the ordinary open string theory without the B field.

The external closed-string tachyon field has been seen to couple only with those components (for the four-dimensional example in Sec.III: X_0, X_1) of the open string coordinates, which are not coupled with the B field. The system breaks down into two dynamically independent subsystems: one consisting of the components (for the four-dimensional example in Sec.III: X_2, X_3) of X coupled with the B field and the other consisting of the remaining components (X_0, X_1) coupled with the closed-string tachyon. This seems to be the reflection of the general feature of the string interaction that the closed string cannot be coupled with the massless vector which is a member of the open string, since there does not exist a vertex of the type "closed-closed-open".

The graviton is also coupled with the noncommutative open string. In the standard weak field approximation of the gravitational field $g_{\mu\nu}(X)$, the vertex operator is given as

$$V = \epsilon_{\mu\nu} J^\mu{}_+(z)U_+(z)U_-(\bar{z})J^\nu{}_-(\bar{z}) = \epsilon_{\mu\nu} J^\mu{}_+(z)e^{2ikX(z,\bar{z})}J^\nu{}_-(\bar{z}), \tag{4.1}$$

where $\epsilon_{\mu\nu}$ is a polarization tensor and required constraints are

$$k^2 = 0, \qquad k\bar{\beta}k = 0. \tag{4.2}$$

In this case there is no σ-dependent coupling factor. The form of (4.1) is the same as in Ref.[10].

For simplicity we have dealt here with the neutral-open string with opposite charges at both ends. We remark that the whole discussion is valid also for the charged-open string with an arbitrary charge at each end [13].

References

[1] H. S. Snyder, Phys.Rev. **71**, 38 (1947)

[2] A. Connes, Noncommutative Geometry, Academic Press 1994.

[3] J. Madore, An Introduction to Noncommutative Geometry and Its Physical Applications, Cambridge University Press, 1999.

[4] A. Connes, M. R. Douglas and A, Schwarz, JHEP 9802,003(1998), hep-th/9711162.

[5] N. Seiberg and E. Witten, JHEP 9909, 032(1999), hep-th/9908142.

[6] L. Castellani, hep-th/0005210.

[7] C. Chu and P. Ho, Nucl. Phys. **B550**, 151 (1999); hep-th/9812219.

[8] F. Ardfaei and M. M. Sheikh-Jabbari, J. High Energy Phys.02, 016(2000); hep-th/9810072; hep-th/9906161. M. M. Sheikh-Jabbari, Phys. Lett. **B455**, 129(1999); hep-th/9901080.

[9] G. Konisi and T. Saito, Prog. Theor. Phys. **55**, 280(1976). T. Saito, Studia Humana et Naturalia (Kyoto Pref. Univ. of Medicine) **22**, 63(1988).

[10] M. Ademollo, A. D'adda, R. D'auria, E. Napolitano, P. Di Vecchia, F. Gliozzi and S. Sciuto, Nucl. Phys. **B77**, 189(1974).

[11] T. Yoneya, Prog. Theor. Phys. **54**, 526(1975).

[12] C. Chu and P. Ho, hep-th/9906192. T. Lee, hep-th/9911140.
W. T. Kim and J. J. Oh, hep-th/9911085. M. M. Sheikh-Jabbari and A. Shirzad, hep-th/9907055.

[13] A. Kokado, G. Konisi and T. Saito, Prog. Theor. Phys. **104**, 1289(2000).